成蹊大学理工学研究報告
J. Fac. Sci.Tech., Seikei Univ.
Vol.47 No.2 (2010)pp.1-6

# On Modeling Ubiquitous Cloud (3): Self-Similarity in Martingale Representation of AIMD TCP Traffic

Shuji Kawasaki*, Jonah Gamba*, Masakazu Higuchi†, Atsushi Koike‡ and Hitomi Murakami‡
Department of Information and Computer Science, Seikei University

*Abstract:* The Ubiquitous Cloud is a concept of large-scale information service network as a social infra-structure. It is featured by real-world context information extraction, user information profiling and self-configuration/-control of network. The objective of the research is to evaluate the network traffic theoretically and thus give a guideline of network design. In this paper, we especially consider interrelating the martingale problem formulation and SS, aiming at obtaining a condition such that the martingale becomes, in certain limiting situation, SS.

*Key–Words:* context-awareness, profiling, self-reconfiguration, spatio-temporal dynamics, long-range dependence, self-similarity, elephant-mice flow.

## 1 Introduction

The Ubiquitous Cloud (UC) is the framework of a ubiquitous network concept that has been advocated since 2003 aiming at providing a theoretical and practical basis of a prospective social infra-structure. The R&D of the concept is conducted in the Ubiquitous network control and administration (Ubila) project of Japan. The project members are experts in the field from companies and universities.

UC is a information service network, as in Fig. 1, with the following features:

- Autonomic extraction of various context information from the real world and coordination of an appropriate service for a user
- Keeping to provide a better service by user information profiling
- Self-reconfiguration of the network

The supposed contents of the context information include the matter of food, clothing and shelter, crime/disaster prevention and rescue, medical/welfare /nursing services, vehicles and ITS (intelligent transport system), economics and business, amenity and favor, etc. The context information of a user in the real-world is detected at any time by sensors deployed everywhere and sent to a server called cloud, which is the brain and controller of the network. The cloud then forms an appropriate service information for the user according to the context or application and sends it to a nearby actuator to provide a real-world service. We call a pair of sensor and actuator a *node*.

Technical elements close to user services level suppose context-awareness /-modeling and location-awareness. These are new themes in the relevant field. Especially for the first one, R&D has just begun and useful results has not obtained yet. In a physical or technological level, on the other hand, suppose IPv6, Heteroscedastic multiplexing, Ad Hoc network, real-time scheduling, etc.

For such large-scale infrastructure network UC, it is important to assess its network traffic theoretically and give a designing guideline. The objective of the research is to obtain the theoretical assessment. Especially we want to know

- how much network capacity is necessary
- how much spatial density of the nodes is necessary
- how long the response time from sensing to actuation is
- how quality of information is evaluated

Here an actuation implies not only the service with physical operation but giving users requested information.

Among the contexts mentioned above, it may be considered that disaster rescue, medical service, care for handicapped persons, etc. are of pubic importance and they often need real-time response. In recent years, on the other hand, a lot of applications in these fields are developed so as to utilize mobile terminals as PDA. Thus a consideration of mobile users in UC may be a problem of high priority.

*Visiting researcher, Dept. Comp. Inform. Sci., Seikei University
†Researcher, Dept. Comp. Inform. Sci., Seikei University
‡Professor, Dept. Comp. Inform. Sci., Seikei University

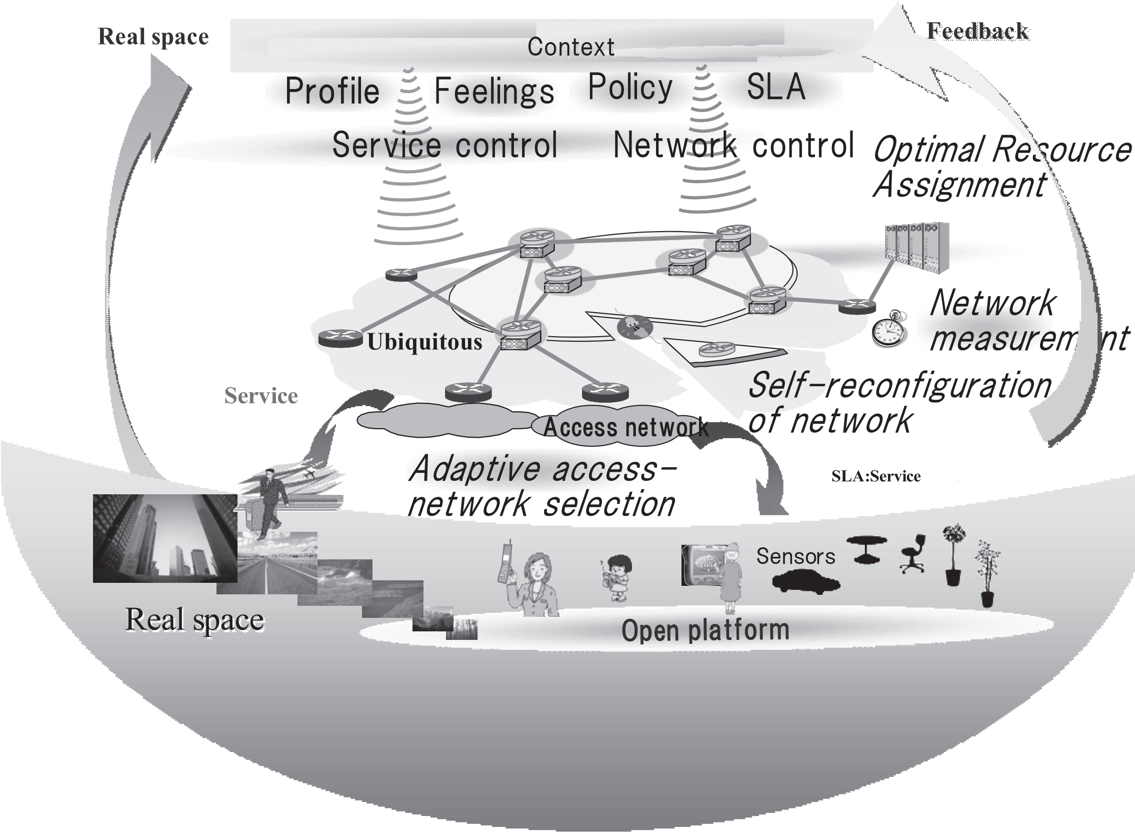


Figure 1: Ubiquitous Cloud Image

In order to realize the network facilities that provides us with sufficiently high quality in a quick response even in the tremendous data flows, it is important to have a good model of the UC and analyze on it, to assess the network performance.

In modeling the UC traffic, the assumed network protocol is crucial in formulation of the network performance. The most widely used and studied protocol so far may be TCP (Transmission Control Protocol) and related ones.

One of the basic model to describe the behavior of a TCP traffic is AIMD (Additive Increase and Multiple Decrease) [1][9]. In particular, Grigorescu and Kang [9] formulated the AIMD traffic as a martingale problem, with the AIMD behavior part as drift component. They considered the traffic subtracted the AIMD drift component may be a martingale component. See [9] for detail.

On the other hand, it is well known that a network traffic shows the marked characteristics called self-similarity (SS) and long-range dependence (LRD) [16][23][26]. Since these properties are caused by a large sum of user sessions and a kind of scaling, the two properties are expected to hold in UC as well.

In this paper, we consider a condition that makes the Grigorescu-Kang martingale representation of the AIMD TCP traffic to have the SS and LRD properties.

The organization of the paper is as follows. In section 2, we survey known results so far and pick up concepts that we will follow. Section 3 is the preliminary consideration for UC traffic modeling, containing some remarks in user mobility formulation. Section 4 then considers the fundamental features that the UC should possess to seek for how to formulate the traffic and what the distinction of UC from classical queueing theory.

## 2 AIMD TCP Model and Its Martingale Representation

The basic behavior of congestion control in a TCP traffic (TCP Reno) can be explained as follows.

When a packet rate of a user session is under a network bottleneck rate, then TCP increase its packet rate. The initial rate is set to 1 and after each receipt of successful *acknowledgment* from receiver, TCP increase 1 segment of *congestion window* size. In the *congestion avoidance* mode, as a result, the packet rate increases by one segment as far as the rate is under the presumed bottleneck.

When the packet rate exceeds the bottleneck, a buffer over-flow occurs at certain portion of router(s) and some packets are lost. In this case, the packet rate is decreased to half of the present, so as to make the packet rate under the bottleneck.

For the TCP congestion control, Grigorescu and Kang [9] considered a traffic model given by the martingale problem through the stochastic differential formula

$$\mathcal{M}_h(t) = h(t, \boldsymbol{x}(t)) - h(0, \boldsymbol{x}(t)) - \int_0^t \Bigg[ \partial_s h(s, \boldsymbol{x}(s-)) + \lambda \sum_{j=1}^{n} \Big( 1 - \zeta_j(s, \boldsymbol{x}(s-)) I_j(s) + \zeta_j(s, \boldsymbol{x}(s-)) D_j(s) \Big) \Bigg] \, ds \tag{1}$$

for a test function $h$ in an appropriate function space. Here

$$I_j(s) = h(s, R_j\boldsymbol{x}(s-)) - h(s, \boldsymbol{x}(s-)) \tag{2}$$

with $R_j\boldsymbol{x} = (x_1, \cdots, x_j{+}a, \cdots, x_n)$ for some $a > 0$, and where

$$D_j(s) = h(s, L_j\boldsymbol{x}(s-)) - h(s, \boldsymbol{x}(s-)) \tag{3}$$

with $L_j\boldsymbol{x} = (x_1, \cdots, \gamma x_j, \cdots, x_n)$ for $0 < \gamma < 1$; $\zeta_j$ is the probability that the congestion occurs at a time instant.

$I_j$ corresponds to increasing packet rate when the rate is under network bottleneck, while $D_j$ to decreasing when the rate exceeds the bottleneck. The martingale formulation of Grigorescu-Kang features the following points:

- The occurrence of congestion is Markov-modulated, i.e. is in accordance with a Poisson process.
- Subtracting the behavior due to the congestion control, the fluctuation of traffic process is considered sufficiently close to white noise.

In this paper, we assume that this martingale problem has a unique solution, i.e. there exists a stochastic process $X$ that makes $\mathcal{M}_h(t)$, when $\boldsymbol{x}$ replaced with $X$, a martingale.

Let $\mathcal{M}_h(t)$ be square integrable and $\mathcal{F}_t$-adapted, for the filtration $\left(\mathcal{F}_t\right)_{t\geq 0}$ generated by a Brownian motion. Then, as is well known, the martingale has the stochastic integral representation

$$\mathcal{M}_h(t) = \mathbb{E}[\mathcal{M}_h(0)] + \int_0^t f(s, \omega) \, dB_s. \tag{4}$$

Let, without loss of generality, $\mathbb{E}[\mathcal{M}_h(0)]$ be 0 for the sake of simplicity. Here $f$ is $\mathcal{F}_t$-adapted and satisfies other conditions (see e.g. [22]).

We assume that the $\mathcal{M}_h(t)$ is Gaussian. In order to show a situation in which this is the case, let us consider that $n$ user sessions are connecting to the network. Let the $j$th session produce the traffic $\tilde{X}_{j,k}$, for discrete time $k = 1, 2, \cdots$. By subtracting a drift component, let us suppose that it becomes a martingale sequence $X_{j,k}$. Then, if the martingale central limit theorem holds, the above Grigorescu-Kang formulation takes place.

For the martingale CLT, one is to validate the so called Lindeberg condition:

$$\frac{1}{s_K^2} \sum_{k=1}^{K} \mathbb{E}[X_{j,k}^2 \mathbb{I}(|X_{j,k}| \geq \varepsilon s_K)] \quad \longrightarrow 0, \tag{5}$$

as $K \to \infty$. Here $s_K^2 = \sum_{k=1}^{K} \mathbb{E}[X_{j,k}^2]$ and $\varepsilon > 0$ is to be taken arbitrarily small.

For the above situation, let us take

$$X_{j,k} = \int_{k-1}^{k} f^{(j)}(s, \omega) \, dB_s^{(j)}. \tag{6}$$

Then, it turns out the Lindeberg condition is indeed satisfied if only there exists a $\delta > 0$ such that $\mathbb{E}[|X_{j,k}|^\delta] < \infty$. In fact,

$$\begin{aligned} &\sum_{k=1}^{K} \mathbb{E}[X_{j,k}^2 \mathbb{I}(|X_{j,k}| \geq \varepsilon s_K)] \\ &\quad = \int_{\varepsilon s_K}^{K} y \, P(|X_{j,k}| \geq y) \, dy \\ &\quad \leq \int_{\varepsilon s_K}^{K} y \, \frac{\mathbb{E}[|X_{j,k}|^\delta]}{y^\delta} \, dy \\ &\quad \leq c \, \mathbb{E}[|X_{j,k}|^\delta] \, K^{2-\delta}. \end{aligned}$$

Since $K \sim s_K^2$, we observe that if $\delta > 1$, the Lindeberg condition is satisfied.

## 3 Establishing Self-similarity in the AIMD TCP Model: FBM Limit Case

In the argument of Taqqu et al.[25] of self-similar process limit of large-sum aggregation of user sessions and time-scaling, each user session is assumed to be a so called ON/OFF source $W(t)$, $t \geq 0$. This $W(t)$ takes value 1 when there is a packet at time $t$ and takes 0 when there is no packet. The length of the ON-periods are i. i. d., those of OFF-periods are i. i. d. as well and the length of ON- and OFF-periods are independent. The ON- and OFF- periods may have different distributions.

Each session has the sequence of packet train given by $\{W^{(m)}(t) | \; t \geq 0\}$. Then, the aggregation of the user sessions over $m$ and time scaling by a factor $T > 0$, they consider the aggregated cumulative packet counts

$$W_M^*(Tt) = \int_0^{Tt} \left[ \sum_{m=1}^{M} W^{(m)}(u) \right] du \tag{7}$$

on the interval $[0, Tt]$. Let $P_{on}$ denote

$$\mathbb{E}[W(t)] = P(t \text{ is on}) = \frac{\mu_1}{\mu_1 + \mu_2}, \tag{8}$$

where $\mu_1, \mu_2 > 0$ are expected values of the ON- and OFF-periods. Then, they showed a stochastic process limit result

$$\lim_{T\to\infty} \lim_{M\to\infty} \frac{W_M^*(Tt) - MTtP_{on}}{T^H L^{1/2}(T) M^{1/2}} \Longrightarrow \sigma B_H(t). \tag{9}$$

This is the first half of their limit result. They have shown, as other half, a non-Gaussian limit as well.

In their argument, a "reward" process $\int_0^t W(u)\, du$ plays a basic roll. In our case, what corresponds to this "reward" is the martingale $\int_0^t f(s,\omega)\, dB_s$. The latter process is stationary-increment. Also, we assume that this martingale is Gaussian. Then, following the proof of their limit result, we are to show the asymptotic growth condition:

$$\begin{aligned} V(t) &= \mathrm{Var}\left[ \int_0^t f(s,\omega)\, dB_s \right] \\ &= \int_0^t \mathbb{E}\big[f^2(s)\big]\, du \\ &\sim t^{2H} L(t) \qquad \text{as} \quad t \to \infty. \end{aligned} \tag{10}$$

If this asymptotic growth condition is valid, then we can show the limit result for our AIMD martingale, as

$$\begin{aligned} &\lim_{T\to\infty} \frac{1}{T^H L^{1/2}(T)} \int_0^{Tt} \left[ \int_0^u f(s,\omega)\, dB_s \right] du \\ &\Longrightarrow \sigma B_H(t), \end{aligned} \tag{11}$$

for some $\sigma > 0$.

Following the proof of Taqqu et al., we consider how to obtain the asymptotic growth. Let $\Gamma_x(u) = \mathbb{E}\big[f^2(s)\big]$. Then

$$\Gamma_x(t) = \int_{\mathbb{R}} f^2(y)\, p(t,x,y)\, dy \tag{12}$$

for the transition probability density $p(t,x,y)$ of the martingale $\int_0^t f(s,\omega)\, dB_s$. Also, let $\gamma_x(t)$ be $d\Gamma_x/dt$.

The method of the proof depends on Laplace transform of $\gamma_x$ and Karamata's Tauberian Theorem [2, Theorem1.6.7].

Let the Laplace transform of $\gamma_x$ be $\hat{\gamma_x}(\xi) = \int_0^\infty e^{-\xi u} \gamma_x(u)\, du$. Then,

$$\begin{aligned} \hat{\gamma_x}(\xi) &= \int_{\mathbb{R}} dy f^2(y) \int_0^\infty e^{-\xi u} \frac{d}{du} p(u,x,y)\, du \\ &= \frac{1}{\xi} \int_{\mathbb{R}} f^2(y)\, \hat{p}(\xi,x,y)\, dy. \end{aligned} \tag{13}$$

Then, since we can write

$$V(t) = \int_0^t \left[ \int_0^u \gamma_x(z)\, dz \right] dy, \tag{14}$$

we have

$$\begin{aligned} \hat{V}(\xi) &= \frac{1}{\xi^2} \hat{\gamma_x}(\xi) \\ &= \frac{1}{\xi^3} \int_{\mathbb{R}} f^2(y)\, \hat{p}(\xi,x,y)\, dy; \end{aligned} \tag{15}$$

Here we want to assume certain asymptotic behavior for $\hat{p}(\xi,x,y)$ as $\xi \to \infty$, so that

$$\hat{V}(\xi) = \xi^{\alpha-3} L(1/\xi), \tag{16}$$

for a *slowly varying function* $L$. A slowly varying function $L$ is a function such that, for all $\lambda > 0$, $L(\lambda x)/L(x) \to 1$ as $x \to \infty$.

If (16) is the case, then

$$V(t) \sim t^{2-\alpha} L(t) \qquad \text{as } t \to \infty \tag{17}$$

is obtained, by the following Karamata's condition for Tauberian Theorem. Thus, we can take $2 - \alpha = 2H$. If we want $1/2 < H < 1$, meaning the LRD, then $0 < \alpha < 1$.

The Karamata's condition for a function $V$ is that it is *slowly decreasing*, i.e.

$$\lim_{\lambda \downarrow 1} \liminf_{x\to\infty} \inf_{t\in[1,\lambda]} \frac{V(tx) - V(x)}{x^{2-\alpha} L(x)} \geq 0. \tag{18}$$

Validating this condition for our $V(t)$ is the present problem. Here we mention the statement of the Karamata's theorem [2, Theorem 1.7.5-6]. It states that if a function $V$ is slowly decreasing, then

$$\hat{V}(\xi) \sim c\xi^{-\rho} L(1/\xi) \quad \text{as } \xi \downarrow 0 \tag{19}$$

implies

$$V(t) \sim ct^{\rho} L(x)/\Gamma(1+\rho) \quad \text{as } x \to \infty. \tag{20}$$

Here $c \geq 0$ and $\rho > -1$.

# 4 conclusion

Towards the UC traffic model formulation, we can consider several point of view in order to make the model distinct from classical queuing theory. Examples of such point of view may be those listed in the beginning of Section 1. Some of the points are complicated version of basic and classical concepts. Among them, formulation along the last two points may have challenging interdisciplinary interests: the connection of SS and LRD property with spatio-temporal process, and statistical mechanics. We would like to explore the formulation along these directions more.

Finally, it will be necessary to perform a verification of the model through a simulation. For this, one may consider simulating the spatio-temporal dynamics by cell auto-maton. As the cell auto-maton is sometimes used for a microscopic behavior simulation to obtain a macroscopic description of the model, we would like to have an equation, like partial differential equation, that describes the spatio-temporal dynamics.

**Acknowledgements:** A part of this work was supported by MEXT Grant-in-Aid for Building Strategic Research Infrastructures.

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